properties of elliptical vibrations as are analogous to those of rectilinear vibrations; and it was in this way that the above rule was discovered. It is analogous (though it scarcely appears so at first sight) to the rule by which, in the theory of Fresnel, the direction of rectilinear vibrations is determined, when the plane of the wave is given.

The Rev. Charles Graves read the first part of a paper on Algebraic Triplets.

The object which he proposes to himself is to frame, for the geometry of three dimensions, a theory strictly analogous to that by which Mr. Warren has succeeded in representing the combined lengths and directions of right lines in a plane. In carrying out this design Mr. Graves has necessarily been led to the consideration of new imaginaries.

For the sake of clearness it will be desirable to take, in the first instance, a brief survey of the fundamental properties of algebraic couplets, depending, as they do, upon the nature of the symbol $\sqrt{-1}$. The correspondence between received notions and the views now put forward will thus be made more apparent.

If we take the binomial or couplet $x + \sqrt{-1} \, . \, y$, in which x and y are real quantities, and multiply it by a similar couplet $x_1 + \sqrt{-1} \, . \, y_1$, the product will likewise be a binomial of the same kind, $x_2 + \sqrt{-1} \, . \, y_2$; and between the constituents of the three couplets there exists the relation

$$(x^2 + y^2) \; (x_1^2 + y_1^2) = x_2^2 + y_2^2. \qquad (a)$$

But couplets may be more readily compared after undergoing a simple transformation. Such an expression as $x + \sqrt{-1} \, . \, y$ may be reduced to the form $re^{\sqrt{-1} . \theta}$ by making $r = \sqrt{x^2 + y^2}$, and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. Hence it appears that if we agree to call r the *modulus* and θ the *amplitude* of a couplet, the following theorems will be true:

1. *If two couplets be multiplied together, the modulus of the product will be equal to the product of the moduli of the factors.*

2. *The amplitude of the product will be equal to the sum of the amplitudes of the factors.*

One of the most important analytical properties of the couplet $x + \sqrt{-1}.y$ consists in this, that the equation

$$x + \sqrt{-1}.y = 0$$

is equivalent to two, viz. $x = 0$ and $y = 0$.

As regards the geometric interpretation of the foregoing results, it is sufficient to observe that the symbol $\sqrt{-1}$ has been explained as denoting rotation through a right angle; whilst the couplet $x + \sqrt{-1}.$ has been taken to represent both the length and the direction of the right line drawn from the origin to the point whose rectangular coordinates are x and y: the length of this right line is obviously r; and it is inclined to the axis of x at an angle equal to θ.

The problem now proposed by Mr. Graves is to assign two distributive symbols, ι and κ, of such a nature that (1) the sum or product of two triplets, $x + \iota y + \kappa z$ and $x_1 + \iota y_1 + \kappa z_1$, shall be itself a triplet of the same form: that (2) there shall be theorems concerning the moduli and amplitudes of triplets, similar to those already enunciated for couplets: that (3) the equation $x + \iota y + \kappa z = 0$ shall be equivalent to the three, $x = 0$, $y = 0$, $z = 0$: and that (4) the symbols ι and κ shall admit of a geometric interpretation analogous to that which has been provided for the symbol $\sqrt{-1}$.

The preceding conditions will be complied with, if we assume ι and κ to be distributive symbols of operation, which, when combined, are subject to the following laws:

$$\iota\kappa(a) = a : \kappa\iota(a) = a : \kappa^2(a) = \iota(a) : \iota^2(a) = \kappa(a).$$

We must, at the same time, agree to regard $\iota(1)$ and $\kappa(1)$ as units absolutely differing in kind from each other and from the real unit. This, in fact, satisfies the third condition. As $\iota^2(a) = \kappa(a)$ we may, for the future, dispense with the symbol κ, and write the triplet in the form $x + \iota y + \iota^2 z$, or more shortly thus (x, y, z).

In the first place, it is evident that the sum or product of two triplets is itself a triplet.

Next, supposing $(x, y, z) \,.\, (x_1, y_1, z_1) = (x_2, y_2, z_2)$ we shall have the modulus of the product equal to the product of the moduli of the factors, if we call the expression $x^2 + y^2 + z^2 + (\iota + \iota^2)(xy + yz + zx)$ the modulus of the triplet (x, y, z). And this *modular* theorem involves in it two others of the same kind, concerning the purely real moduli, $x + y + z$, and $x^2 + y^2 + z^2 - xy - yz - zx$. According as we bring the triplet into different forms by changing the variables in it, there will be either two theorems relating to moduli, and one relating to amplitudes, or one modular theorem, and two concerning amplitudes.

For the purpose of geometric interpretation let us suppose the three positive portions of the axes of rectangular coordinates to meet the surface of a sphere, whose centre is at the origin, in the points X, Y, Z, through which a small circle of the sphere is described, and let us give the name of *symmetric axis* to that diameter of the sphere, which passes through the poles of this circle. Now if we conceive the real unit placed on the axis of x, $\iota(1)$ on the axis of y, and $\iota^2(1)$ on the axis of z, we may interpret the symbol ι by saying that it denotes a conical rotation round the symmetric axis through an angle of 120 degrees. Three such operations, executed successively on the real unit, will bring it back to its original position on the axis of x. This is in accordance with the equations

$$\iota\kappa(a) = \kappa\iota(a) = \iota^3(a) = a:$$

in virtue of which we may regard $\iota(1)$ as a purely imaginary cube root of positive unity.

Mr. Graves mentioned that, since he had obtained permission to read the present paper, Sir William R. Hamilton had kindly communicated to him the abstract and the proof sheets of a memoir by Professor De Morgan, on Triple Algebra. That paper contains the discussion of a system of triplets, which is most closely connected with the one now proposed: the only difference being that Professor De Morgan uses what are in fact new imaginary cube roots of *negative* unity.

Mr. Graves thinks that in the interpretation and generalization of his results he has met with greater success; but he fully concedes to Professor De Morgan the prior possession of what must be looked upon as fundamental in this theory, the conception of symbols which act upon each other in the same manner as the imaginary cube roots of unity. Mr. Graves also stated that his brother, John T. Graves, Esq., had anticipated him in the idea of using cube roots of positive unity in the constitution of algebraic triplets.

The remaining portion of the paper, having reference chiefly to the interpretation of the formulæ obtained in the multiplication of triplets, was postponed until the next meeting of the Academy.

Mr. George Yeates presented a tabular Return of the Observations made by him with Barometer, Thermometer, and Rain Gauge, at his residence, near Portobello, County of Dublin, during the year ending 31st December, 1844.—(See *Appendix*, No. II.)

DONATIONS.

An Essay on Aerial Navigation. By Joseph M'Sweeny, M. D. Presented by the Author.

Archæologia, Vol. XXX., and *Index* to Vols. XVI. to XXX., inclusive. Presented by the Society of Antiquaries of London.

J. H. R. Mott's Advice and Instructions for playing the